

Which quantum states are dual to classical spacetimes?

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Abstract

It is commonly accepted that states in a conformal field theory correspond to classical spacetimes with Anti-de-Sitter asymptotics. In this essay, we argue that such states should be coherent in the large- N limit, and show implications in the spacetime emergence mechanism. In particular, we argue that the microstates that compose a black hole (entangled) state in the Van Raamsdonk description cannot be interpreted as classical geometric configurations. Therefore, the conclusion is that care should be taken to interpret (micro)states in the gravity side, and that quantum coherence plays an important role in the description of the holographic emergence phenomenon.

Introduction

The AdS/CFT correspondence represents the paradigmatic case of gravity/gauge duality where spacetime with fixed (AdS) asymptotics can be defined as emergent from an ordinary quantum field theory defined on its conformal boundary [1]. However, we do not understand the mechanism of this emergence in depth.

It is widely accepted that states in the CFT Hilbert space are dual to classical asymptotically AdS (aAdS) spacetimes but we do not know yet which aspects of the classical geometries are encoded in the states or how to read off such aspects. Moreover, it is unclear which CFT states actually are dual to some type of classical geometry. An illuminating observation was made by Van Raamsdonk some years ago [2], who argued that classically connected spacetimes correspond to *entangled* states in the CFT, and furthermore, that they are a quantum superposition of basis states supposedly dual themselves to some kind of aAdS spacetimes. This set up has been used in further developments [3, 4].

This essay is devoted to study the (disentangled) microstates that compose the spacetime geometry as a quantum superposition, and whether they are dual to geometries with usual classical properties. We will use recent results [5], which are in line with previous suggestions [6], to argue that quantum coherence is also an essential ingredient for the emergence of classical spacetimes, and consequently show that the microstates that form an AdS Black Hole do not correspond to smooth geometric backgrounds in general.

The emergent AdS Black Hole

The standard interpretation is that the exact bulk geometry AdS_{d+1} corresponds to the fundamental state $|0\rangle$ of the CFT Hilbert space \mathcal{H} defined on its conformal boundary $S^d \times \mathbb{R}$, and general classical aAdS spacetimes should be dual to certain excited states.

Let us consider now two (non-interacting) identical copies of this CFT (labeled by a subindex 1, 2). The asymptotically AdS_{d+1} spacetime with an eternal black hole corresponds to the (entangled) state [7]:

$$|\Psi(\beta)\rangle\rangle = \sum_n \frac{e^{-\frac{\beta}{2} E_n}}{Z^{1/2}} |E_n\rangle_1 \otimes |E_n\rangle_2 \in \mathcal{H}_1 \otimes \mathcal{H}_2, \quad \beta \equiv (k_B T)^{-1}, \quad (1)$$

where the $|\dots\rangle\rangle$ denotes a ket that belongs to the product of the CFT Hilbert spaces $\mathcal{H}_1 \otimes \mathcal{H}_2$, the $|E_n\rangle$ are a complete basis of eigenstates of the CFT Hamiltonian H , and E_n are the eigenvalues. This describes a thermal state of the CFT system at temperature T in the *thermofield dynamics* (TFD) formalism [8, 9, 10]. Recall also that the state (1) is invariant under the action of the combination $H_1 - H_2$.

One of the arguments of [2] was precisely based in the structure of this state, which describes a classically connected spacetime (fig. 1a) with two aAdS regions causally separated by an event horizon [11] using a quantum superposition of states $|E_n\rangle_1 \otimes |E_n\rangle_2$ (fig. 1b), which, *if* have a classical geometric dual, they should correspond to disconnected aAdS spacetimes.

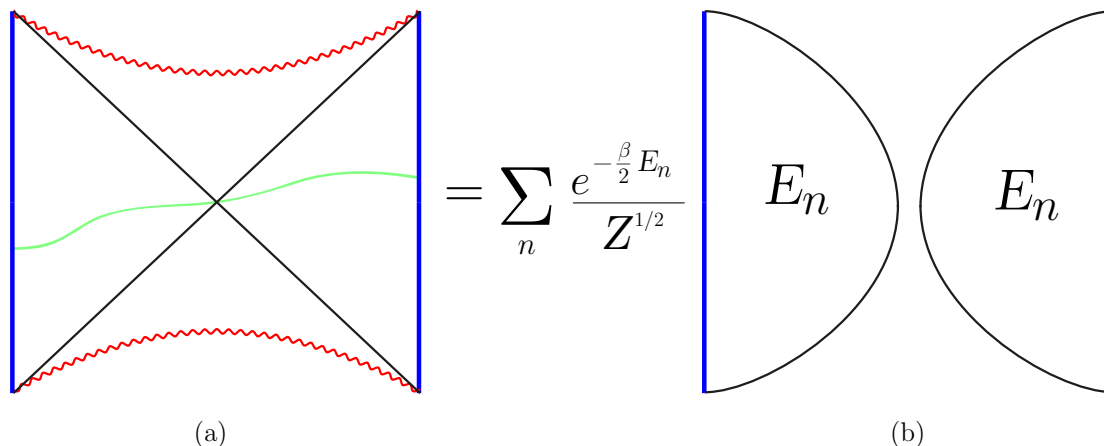


Figure 1: (a) Penrose diagram of a maximally extended AdS-black hole. The green line is a connected spacial slice representing the entanglement between the CFT's. (b) We schematically show the interpretation of [2], where the resulting state (1) is a linear combination of states $|E_n\rangle_1 \otimes |E_n\rangle_2$ supposedly dual to aAdS spacetimes. The blue lines represent the non-interacting CFT theories on the two asymptotic boundaries.

The argument for it is that disentangled states $|\Psi\rangle_1 \otimes |\Upsilon\rangle_2 \in \mathcal{H}_1 \otimes \mathcal{H}_2$ of two systems CFT₁ and CFT₂ that do not interact in any way, must describe two completely separate physical systems. Then if $|\Psi\rangle_1$ is dual to one aAdS spacetime and $|\Upsilon\rangle_2$ is dual to some other spacetime, the product state must be a geometry dual to the disconnected pair of spacetimes (see Fig 1b). In particular, the ground state is $|0\rangle_1 \otimes |0\rangle_2$, that correspond to two disconnected globally AdS spacetimes [2].

In order to turn this interpretation useful for holographic quantum gravity one should better understand which is the geometric content of these (micro)states. Below, we will propose a suitable definition of the states in each CFT copy that are holographically dual to aAdS spacetimes and show that they could not be the microstates $|E_n\rangle_1 \otimes |E_n\rangle_2$, and consequently, one should find another way of representing the $|E_n\rangle$'s in the gravity side, perhaps in terms of other type of microscopic geometry rather than a smooth d -dimensional spacetime.

CFT states with spacetime dual

The prescription [12, 13] allows the calculation of time ordered n -point correlation functions of local CFT operators \mathcal{O} in AdS/CFT (fig. 2a). It can be understood as the real time version of the standard prescription [14, 15] (fig. 2b), and corresponds to the geometry shown in figure 3a.

$$\langle 0 | e^{-i \int_{\partial \mathcal{M}_L} \mathcal{O} \phi_L} | 0 \rangle = e^{i S_L^0[\phi_L; \phi_{\Sigma-}, \phi_{\Sigma+}] - S_-^0[0; \phi_{\Sigma-}] - S_+^0[0; \phi_{\Sigma+}]} . \quad (2)$$

The lhs gives the generating function of time ordered correlation functions of a scalar

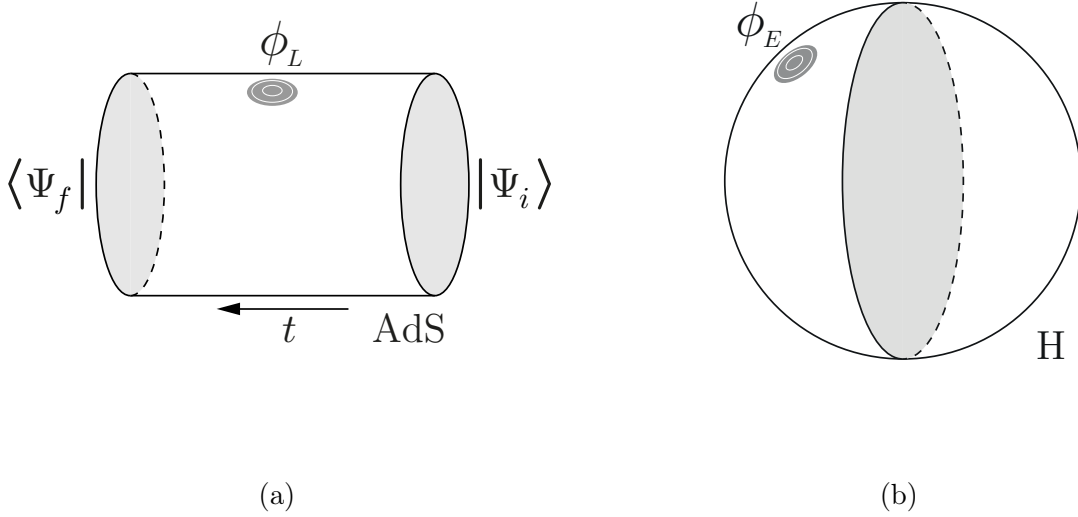


Figure 2: (a) Lorentzian AdS with boundary condition ϕ_L . We also schematically depict the initial and final states. (b) Euclidean AdS (hyperbolic space).

operator \mathcal{O} in a Lorentzian CFT that lives in the timelike conformal boundary $\partial_r \mathcal{M}_L = S^d \times \mathbb{R}$ of the spacetime \mathcal{M}_L . In the rhs, $S_L^0[\phi_L; \phi_{\Sigma^-}, \phi_{\Sigma^+}]$ is the Lorentzian on-shell action for a bulk field Φ_L which takes boundary values ϕ_{Σ^\pm} on the spacelike boundaries $\Sigma^\pm \equiv \partial_t \mathcal{M}_L$ and ϕ_L over $\partial_r \mathcal{M}_L$. The exponents $S_\pm^0[0; \phi_{\Sigma^\pm}]$ are the bulk field on shell actions on the Euclidean sections \mathcal{M}_\pm for boundary values $\phi_\pm = 0$ on $\partial_r \mathcal{M}_\pm$ and ϕ_{Σ^\pm} on Σ^\pm . It is worth noticing that (2) implicitly assumes the bulk fields, and its conjugated momenta, to be continuous through the Σ^\pm gluing surfaces.

This recipe follows from a complete quantum treatment where the r.h.s. is a path integral, and where one can also consider *excited* CFT initial/final states by giving smooth non-vanishing boundary conditions ϕ_\pm over $\partial_r \mathcal{M}_\pm$ for all finite value of the euclidean time¹ (see refs [5] and [12] for details). In this context, in ref. [5], it was explicitly shown that the (initial) excited CFT states are precisely given by

$$|\Psi^{\phi_-}\rangle = e^{-\int_{\partial_r \mathcal{M}_-} \mathcal{O} \phi_-} |0\rangle. \quad (3)$$

These states, projected on a complete bulk configuration basis ϕ_{Σ^\pm} on spacelike surfaces Σ^\pm correspond to wave functionals that can be expressed as euclidean path integrals in the gravity side,

$$\Psi^{\phi_-}[\phi_{\Sigma^-}] \equiv \int [\mathcal{D}\Phi]_{(\phi_{\Sigma^-}, \phi_-)} e^{-S_-[\Phi]}, \quad (4)$$

with boundary conditions ϕ_{Σ^-}, ϕ_- on $\Sigma^-, \partial_r \mathcal{M}_-$ respectively. Moreover, in the large- N limit one can use the saddle point approximation and they can be associated to a single classical

¹They vanish at $\tau = \pm\infty$, represented as green dots in fig. 3b.

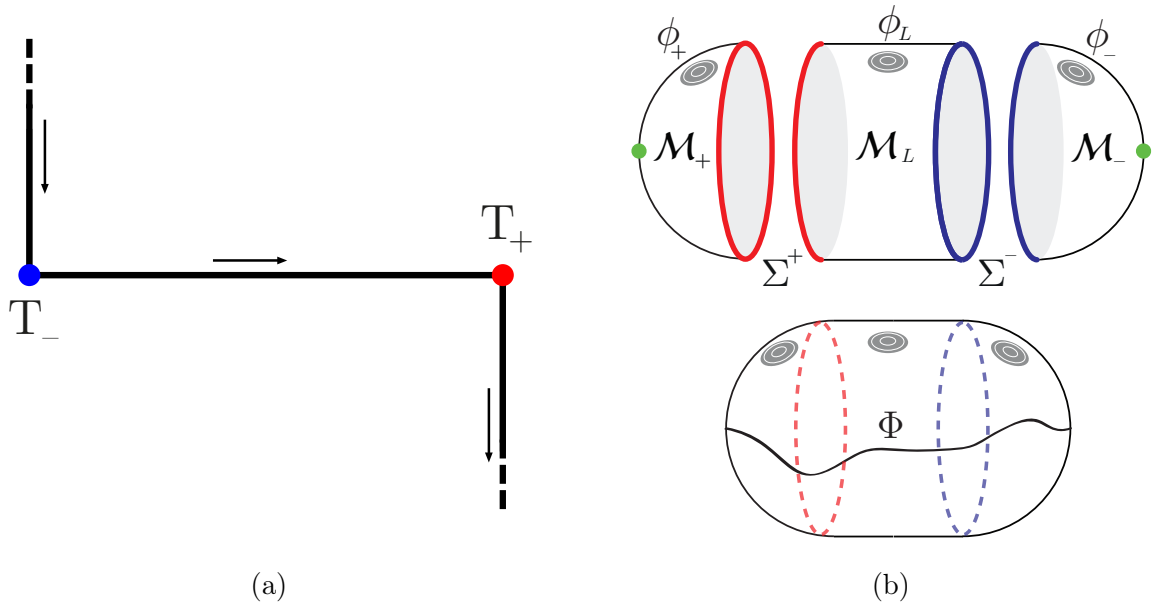


Figure 3: (a) CFT contour in complex t -plane showing real time interval (T_-, T_+) . (b) Geometric dual to the contour obtained by gluing together Lorentzian \mathcal{M}_L and Euclidean sections (two halves) of the AdS spacetime \mathcal{M}_\pm . We depict the gluing surfaces Σ^\pm and the green points correspond to $\tau = \pm\infty$. On the bottom we show the glued geometry with a generic classical configuration.

solution for the bulk fields as

$$\Psi^{\phi_-}[\phi_{\Sigma^-}] \sim e^{-S_-^0[\phi_{\Sigma^-}, \phi_-]} . \quad (5)$$

This generalizes the Hartle-Hawking construction of wave functionals [16] to excited states in AdS quantum gravity [5, 12]. Obviously this analysis can be extended to include the gravitational degrees of freedom, although in our arguments here we consider only a scalar field that probes the spacetime. For instance, the existence of classical fields implies that the background manifold \mathcal{M} should have certain classical properties such as connectivity and differentiability.

The conclusion here is that these are the states that can be associated, to classical geometric configurations. For quantum bulk theory, there is be *a set* of geometries/paths associated, as the space to sum over in (4). Then in the large- N limit, there is a unique classical solution describing an (euclidean) spacetime with matter fields $(g_{\mu\nu}, \Phi, \dots)$.

The Large- N limit and quantum coherence

Taking the large- N limit in the above expression (4), the Euclidean action for Φ becomes gaussian [17] and it also decouples from gravity, since the Newton constant $G_N \sim 1/N^2$.

For free fields, another standard prescription [17, 18, 19] identifies the dual CFT operators

with the boundary value of a canonically quantized field in AdS, $\hat{\Phi}(t, r, \Omega)$, as

$$\hat{\mathcal{O}}(t, \Omega) \equiv \lim_{r \rightarrow \infty} r^\Delta \hat{\Phi}(t, r, \Omega) = \sum_k a_k^\dagger F_k^*(t, \Omega) + a_k F_k(t, \Omega), \quad (6)$$

where $F_k(t, \Omega) \equiv \lim_{r \rightarrow \infty} r^\Delta f_k(t, r, \Omega)$ defines a basis of functions on the conformal boundary and f_k are the basis of normalizable solutions of the e.o.m. $(\square - m^2)\Phi = 0$ labeled by k .

Demanding consistency between both prescriptions, we conclude that in the large-N limit, the state (3) is *coherent* and can be represented as

$$|\Psi^\phi\rangle \propto e^{\sum_k \lambda_k a_k^\dagger} |0\rangle, \quad (7)$$

in the (bulk) Hilbert-Fock space \mathcal{H}_{AdS} . The same analysis is possible for linear perturbations of the background metrics $g_{\mu\nu}$.

This expression has been checked through explicit holographic computations of correlation functions and inner products [5]. These states are *eigenstates* of the annihilation operators a_k are their eigenvalues are precisely given by

$$\lambda_k = - \int_{(-\infty, 0] \times S^d} d\tau d\Omega F_k^*(-i\tau, \Omega) \phi^-(\tau, \Omega).$$

On the other hand, the (excited) states $|E_k\rangle_1 \otimes |E_k\rangle_2$ of the basis in which the Black Hole entangled state is expanded in the Van Raamsdonk setup, correspond to eigen-states of the Hamiltonian operator $H = H_\Phi + H_{gravity} + \dots + o(1/N)$, where

$$: H_\Phi := \sum_k \epsilon_k a_k^\dagger a_k, \quad (8)$$

and \dots denote terms associated with other (matter) fields.

So there are states in the basis that express as $|E_k\rangle_1 \sim (a_k^\dagger)^n |0\rangle_\phi \otimes |0\rangle_{gravity} \otimes |\dots\rangle$, where $|0\rangle_{gravity}$ denotes the exact globally AdS spacetime, or any other aAdS solution of vacuum Einstein equations². The key point in our argument is that this type of states (except the vacuum) are not eigenstates of a_k since these operators do not commute with H_Φ . Therefore, these states cannot be associated to classical configurations/solutions in the sense explained above. In other words, there are states $|E_k\rangle_1 \otimes |E_k\rangle_2$ in the $CFT_1 \otimes CFT_2$ Hilbert space which *are not* dual to any classical spacetime with classical fields.

To enforce the starting point of our argument, eq. (7), let us recall the important property of these states that clearly expresses the notion of correspondence with classical configurations and might be seen as a suitable definition for geometric dual states. Notice that the expectation value of the quantized field $\hat{\Phi}$, coincides with a classical solution:

$$\langle \Psi | \hat{\Phi} | \Psi \rangle = \Phi_{cl}. \quad (9)$$

²As argued above, this state can also be associated to some other background (euclidean) metric that minimizes the gravitational action, and then give wave functions (3) in the large-N approximation.

This is satisfied for coherent states but fails for states $(a_k^\dagger)^n |0\rangle_\phi$ with $n > 0$. Moreover, only for coherent states this property can be generalized to any functional $\mathcal{F}[\hat{\Phi}]$ of the quantized fields and its derivatives (e.g. the Hamiltonian H_Φ), which, with the proper normal ordering, coincides with the same functional valued on a classical solution:

$$\langle \Psi | : \mathcal{F}[\hat{\Phi}] : | \Psi \rangle = \mathcal{F}[\Phi_{cl}]. \quad (10)$$

The eternal black hole as coherent state

Since the eternal AdS Black Hole is a geometric state of the doubled CFT theory, schematically denoted as $\text{CFT}^2 (\equiv \text{CFT}_1 \otimes \text{CFT}_2)$ [3], this indeed fits into the proper extension of the notion of coherent state.

Thermal Bogoliubov's transformations G_β are formally unitary and canonical in the sense of preserving the canonical commutation relations and the norm of the states in finite volume systems, so then, the thermal TFD state (dual to a AdS black hole) (1) is an eigenstate (with eigenvalue = 0) of the thermal annihilation operators

$$a_k^{(1)}(\beta) \equiv a_k^{(1)} - e^{-\beta\epsilon_k/2} a_k^{\dagger(2)} = G_\beta a_k^{(1)} G_\beta^\dagger \quad (11)$$

$$a_k^{(2)}(\beta) \equiv a_k^{(2)} - e^{-\beta\epsilon_k/2} a_k^{\dagger(1)} = G_\beta a_k^{(2)} G_\beta^\dagger \quad (12)$$

then, the equations $a_k^{(i)}(\beta) |\Psi(\beta)\rangle = 0$, $i = 1, 2$ are solved by the state [3]:

$$|\Psi(\beta)\rangle_{(\Phi)} = G_\beta |0\rangle = Z_\Phi^{-1/2} \prod_k \left[e^{(e^{-\beta\epsilon_k/2}) a_k^{\dagger(1)} a_k^{\dagger(2)}} \right] |0\rangle. \quad (13)$$

This is the state (1) represented in the scalar field sector of doubled AdS Hilbert-Fock space, in the same sense that the states (7) in a single CFT theory³. This state is also invariant under the action of the extended Hamiltonian $H_\Phi^{(1)} - H_\Phi^{(2)}$ [8, 9, 10].

The Hawking-Page quantum transition

Let us finally point out an implication of our arguments on the Hawking-Page phase transition [20, 21]. It can be described in a quantum mechanical way as a critical behavior of the quantum amplitude and standard quantum collapse [3], in line with the Van Raamsdonk interpretation. Since the AdS-BH spacetime is described by the state (1) in CFT^2 ; which at low temperatures (compared with the AdS scale $\beta \sim R_{AdS}$) reads as

$$|\psi(\beta)\rangle = \frac{e^{-\frac{\beta}{2}E_0}}{Z^{1/2}} |0\rangle_1 \otimes |0\rangle_2 + \sum_{n \neq 0} \delta_n(\beta) |E_n\rangle, \quad (14)$$

³Notice that these can be also interpreted as “eigenstates” of both (zero temperature) operators a_k^i , $i = 1, 2$ in the following generalized sense $a_k^i |\Psi(\beta)\rangle = \hat{\lambda}_k^i |\Psi(\beta)\rangle$, $\forall i = 1, 2$ where $\hat{\lambda}_k^i$ is not a c-number, but rather an operator that acts non trivially on the space $\mathcal{H}_{j \neq i}$ since $[a_k^i, \hat{\lambda}_k^i] = 0$.

where $|E_n\rangle\rangle$ are states orthogonal to $|0\rangle_1 \otimes |0\rangle_2$, and $|\delta_k(\beta)|^2 \ll |Z^{-1} e^{-\beta E_0}|$. as $\beta > E_0^{-1} \sim R$ (here R is the radius of curvature of the AdS space). So at low temperatures, the probability of collapsing this state into two disconnected global AdS spaces (see fig. 1a) is very high, compared with other states.

However this description must be complemented with our previous arguments since it should be explained why, for the high temperature regime where the amplitudes are comparable, the system cannot collapse into some of the excited states $|E_n\rangle_1 \otimes |E_n\rangle_2$ upon measurements, giving place to other transitions. The reason is that such states could not be described in semi-classical regimes (large- N) in terms of fields on classical spacetime geometries. The single geometric states that can be probed with fields on classical spacetimes are precisely the black hole state and the gravity ground state $|0\rangle_1 \otimes |0\rangle_2$ that describes two disconnected copies of AdS, or both spacetimes with matter fields in thermal equilibrium (e.g eq. (13))⁴.

Conclusions

We have argued that the $d + 1$ dimensional spacetime geometries containing classical (scalar) fields are dual to the coherent states of a CFT defined on the conformal boundary, which noticeably reduces the possibilities for the spacetime geometry to global AdS, any other aAdS solution of the vacuum Einstein equation, and linear perturbations on these backgrounds [5]. As a first consequence of this, we observed that the basis microstates of the linear superposition (13) should not be generically interpreted as classical spacetimes in the gravity side, but perhaps in terms of some type of non-standard (quantum) geometry.

The next class of asymptotically AdS spacetimes are obtained by entangling the CFT with another identical (non-interacting) copy [2, 3], which are also coherent states (see (13)) that correspond to general (charged, rotating, ...) AdS Black Holes. It is fascinating that a kind of change of the spacetime topology seems to correspond to entangling two dual quantum field theories.

Notice that this essay might be interpreted as a sort of holographic argument in favor of the no-hair conjecture in asymptotically AdS spacetimes, since according to it, it seems to be impossible to build up classical static solutions apart from globally AdS or AdS Black Holes being background for static classical field configurations, which should be simultaneously associated to eigenstates of H (or $H_1 - H_2$) and to coherent states.

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Note Added: This essay was written for the Gravity Research Foundation 2016 Awards for Essays on Gravitation. In the present version we added the “The Hawking-Page quantum transition” section based on the work [3]. We have recently become aware of the work [22] where the discussion on the Hawking Page transition have similar aspects.

⁴The state $|0\rangle_1 \otimes |0\rangle_2$ here only expresses the gravity part. For simplicity, we have omitted (coherent) excitations of Φ and other matter fields.

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